Lösung zur Übung zur partiellen Integration

1. $\int_1^3 x \cdot e^{7x-3} dx$ u(x) = x u'(x) = 1 v'(x) = e^{7x-3} v(x) = $\frac{1}{7} e^{7x-3}$	= $\left[x \cdot \frac{1}{7} e^{7x-3}\right]_1^3 - \int_1^3 1 \cdot \frac{1}{7} e^{7x-3} dx$ = $[\frac{3}{7} \cdot e^{18} - \frac{1}{7} \cdot e^4] - \left[\frac{1}{49} e^{7x-3}\right]_1^3$ = $[\frac{3}{7} \cdot e^{18} - \frac{1}{7} \cdot e^4] - [\frac{1}{49} e^{18} - \frac{1}{49} \cdot e^4]$ ≈ 26.799.980
2. $\int_0^4 3x \cdot e^{2x} dx$ u(x) = 3x u'(x) = 3 v'(x) = e^{2x} v(x) = $\frac{1}{2} e^{2x}$	= $\left[3x \cdot \frac{1}{2} e^{2x}\right]_0^4 - \int_0^4 3 \cdot \frac{1}{2} e^{2x} dx$ = $[\frac{3}{2} \cdot 4 \cdot e^8 - \frac{3}{2} \cdot 0 \cdot e^0] - \left[\frac{3}{4} e^{2x}\right]_0^4$ = $[6 \cdot e^8] - [\frac{3}{4} e^8 - \frac{3}{4} \cdot e^0]$ ≈ 15650,8
3. $\int_{-1}^2 (4x+1) \cdot e^{3x+1} dx$ u(x) = 4x+1 u'(x) = 4 v'(x) = e^{3x+1} v(x) = $\frac{1}{3} e^{3x+1}$	= $\left[(4x+1) \cdot \frac{1}{3} e^{3x+1}\right]_{-1}^2 - \int_{-1}^2 4 \cdot \frac{1}{3} e^{3x+1} dx$ = $[3 \cdot e^7 - (-1) \cdot e^{-2}] - \left[\frac{4}{9} e^{3x+1}\right]_{-1}^2$ = $[3 \cdot e^7 + e^{-2}] - [\frac{4}{9} e^7 - \frac{4}{9} \cdot e^{-2}]$ ≈ 2802,7
4. $\int_1^5 \ln(x) dx$ = $\int_1^5 \ln(x) \cdot 1 dx$ u(x) = ln(x) u'(x) = $\frac{1}{x}$ v'(x) = 1 v(x) = x	= $[\ln(x) \cdot x]_1^5 - \int_1^5 \frac{1}{x} \cdot x \, dx$ = $[\ln(x) \cdot x]_1^5 - \int_1^5 1 \, dx$ = $[\ln(5) \cdot 5 - \ln(1) \cdot 1] - [x]_1^5$ = $\ln(5) \cdot 5 - 4$ ≈ 4,047
5. $\int_1^3 x \cdot \ln(x) dx$ u(x) = ln (x) u'(x) = $\frac{1}{x}$ v'(x) = x v(x) = $\frac{1}{2}x^2$	= $\left[\ln(x) \cdot \frac{1}{2} x^2\right]_1^3 - \int_1^3 \frac{1}{x} \cdot \frac{1}{2} x^2 \, dx$ = $\left[\ln(x) \cdot \frac{1}{2} x^2\right]_1^3 - \int_1^3 \frac{1}{2} x \, dx$ = $[\ln(3) \cdot 4{,}5 - \ln(1) \cdot 0{,}5] - \left[\frac{1}{4} x^2\right]_1^3$ = $\ln(3) \cdot 4{,}5 - [\frac{9}{4} - \frac{1}{4}]$ ≈ 2,944

6. $\int_{-1}^{2\pi} x \cdot \cos(x)dx$ u(x) = x u'(x) = 1 v'(x) = cos(x) v(x) = sin(x)	$= [x \cdot \sin(x)]_{-1}^{2\pi} - \int_{-1}^{2\pi} 1 \cdot \sin(x)\,dx$ $= [2\pi \cdot sin(2\pi) - (-1) \cdot sin(-1)] - [-\cos(x)]_{-1}^{2\pi}$ $= \sin(-1) - [-\cos(2\pi) + \cos(-1)]$ $\approx -0{,}832$
7. $\int_{0}^{\pi} (4x - 5) \cdot \sin(x)dx$ u(x) = 4x−5 u'(x) = 4 v'(x) = sin(x) v(x) = −cos(x)	$= [(4x-5) \cdot (-\cos(x)]_0^{\pi} - \int_0^{\pi} 4 \cdot (-\cos(x))\,dx$ $= [(4\pi - 5) \cdot (-\cos(\pi)) - (-5) \cdot (-\cos(0))] - [-4\sin(x)]_0^{\pi}$ $= [(4\pi - 5) \cdot (-(-1)) - (-5) \cdot (-1)] - [-4\sin(\pi) + 4\sin(0)]$ $= [(4\pi - 5) - 5] - 0$ $\approx 2{,}567$
8. $\int_{1}^{3} x^2 \cdot e^{4x-3}dx$ u(x) = x² u'(x) = 2x v'(x) = e^{4x-3} v(x) = $\frac{1}{4}e^{4x-3}$ u(x) = $\frac{1}{2}x$ u'(x) = $\frac{1}{2}$ v'(x) = e^{4x-3} v(x) = $\frac{1}{4}e^{4x-3}$	$= \left[x^2 \cdot \frac{1}{4} \cdot e^{4x-3}\right]_1^3 - \int_1^3 2x \cdot \frac{1}{4} \cdot e^{4x-3}dx$ $= (\frac{9}{4} \cdot e^9 - \frac{1}{4} \cdot e^1) - \int_1^3 \frac{1}{2}x \cdot e^{4x-3}dx$ $\approx 18231{,}3 - [\left[\frac{1}{2}x \cdot \frac{1}{4} \cdot e^{4x-3}\right]_1^3 - \int_1^3 \frac{1}{2} \cdot \frac{1}{4} \cdot e^{4x-3}dx]$ $= 18231{,}3 - [(\frac{3}{8} \cdot e^9 - \frac{1}{8} \cdot e^1) - \left[\frac{1}{8} \cdot \frac{1}{4} \cdot e^{4x-3}\right]_1^3]$ $\approx 18231{,}3 - [3038{,}32 - (\frac{1}{32} \cdot e^9 - \frac{1}{32} \cdot e^1)]$ $\approx 18231{,}3 - 3038{,}32 + 253{,}139 \approx 15446{,}1$
9. $\int_{0}^{\pi} x^2 \cdot \cos(x)dx$ u(x) = x² u'(x) = 2x v'(x) = cos(x) v(x) = sin(x) u(x) = 2x u'(x) = 2 v'(x) = sin(x) v(x) = −cos(x)	$=[x^2 \cdot \sin(x)]_0^{\pi} - \int_0^{\pi} 2x \cdot \sin(x)\,dx$ $= [\pi^2 \cdot \sin(\pi) - 0] - [2x \cdot (-\cos(x)]_0^{\pi} - \int_0^{\pi} 2 \cdot (-\cos(x))dx]$ $= 0 - [2\pi \cdot (-\cos(\pi)) - 0 - [-2\sin(x)]_0^{\pi}]$ $= 2\pi \cdot \cos(\pi) - 0$ $\approx -6{,}283$