Lösungen zu den Übungen zur Integration mit Substitution

$$\int_a^b f(g(x)) \cdot g'(x) = \int_{g(a)}^{g(b)} f(t)dt$$

1. $\int_0^2 4x^3 \cdot e^{x^4+2}dx$ f(x) = e^x g(x) = $x^4 + 2$	= $\int_2^{18} e^t dt$ = $[e^t]_2^{18}$ = $e^{18} - e^2$ ≈ 65.659.961,75
2. $\int_{-1}^2 x^2 \cdot e^{x^3-8}\, dx$ f(x) = e^x g(x) = $x^3 - 8$	$= \frac{1}{3} \cdot \int_{-1}^2 3x^2 \cdot e^{x^3-8}dx = \frac{1}{3} \cdot \int_{-9}^0 e^t dt$ $= \frac{1}{3} \cdot [e^t]_{-9}^0 = \frac{1}{3} \cdot (e^0 - e^{-9}) \approx 0{,}33$
3. $\int_{-1}^3 \frac{4}{4x+6} dx$ f(x) = x^{-1} g(x) = 4x + 6	$= \int_{-1}^3 4 \cdot (4x+6)^{-1}dx = \int_2^{18} t^{-1}dt$ $= [\ln(t)]_2^{18}$ = ln(18) – ln(2) ≈ 2,197
4. $\int_1^4 \frac{4x}{(6x^2-4)^4} dx$ f(x) = x^{-4} g(x) = $6x^2 - 4$	$= \frac{1}{3} \cdot \int_1^4 \frac{12x}{(6x^2-4)^4} dx = \frac{1}{3} \cdot \int_2^{60} t^{-4}dt$ $= \frac{1}{3} \cdot \left[-\frac{1}{3}t^{-3}\right]_2^{60} = -\frac{1}{9} \cdot 60^{-3} + \frac{1}{9} \cdot 2^{-3}$ ≈ 0,01389
5. $\int_0^3 \frac{4x}{\sqrt{2x^2+8}} dx$ f(x) = $x^{-0,5}$ g(x) = $2x^2 + 8$	$= \int_8^{26} t^{-0,5}dt$ $= [2 \cdot t^{0,5}]_8^{26} = 2 \cdot 26^{0,5} - 2 \cdot 8^{0,5}$ ≈ 4,54
6. $\int_{-1}^0 \frac{3x^2}{\sqrt{2x^3+8}} dx$ f(x) = $x^{-0,5}$ g(x) = $2x^3 + 8$	$= \frac{1}{2} \cdot \int_{-1}^0 \frac{6x^2}{\sqrt{2x^3+8}} dx = \frac{1}{2} \cdot \int_6^8 t^{-0,5}dt$ $= \frac{1}{2} \cdot [2 \cdot t^{0,5}]_6^8 = 8^{0,5} - 6^{0,5} \approx 0{,}379$
7. $\int_{-2}^1 \frac{e^x}{6e^x+2} dx$ f(x) = x^{-1} g(x) = $6e^x + 2$	$= \frac{1}{6} \cdot \int_{-2}^1 \frac{6e^x}{6e^x+2} dx \approx \frac{1}{6} \cdot \int_{2,81}^{18,31} t^{-1}dt$ $= \frac{1}{6} \cdot [\ln(t)]_{2,81}^{18,31} = \frac{1}{6} \cdot \ln(18{,}31) - \frac{1}{6} \cdot \ln(2{,}81)$ ≈ 0,31
8. $\int_1^2 x^3 \cdot \ln(3x^4+4)dx$ f(x) = ln(x) g(x) = $3x^4 + 4$	$= \frac{1}{12} \cdot \int_1^2 x^3 \cdot \ln(3x^4+4)dx = \frac{1}{12} \cdot \int_7^{52} \ln(t)dt$ $= \frac{1}{12} \cdot [t \cdot \ln(t) - t]_7^{52}$ $= \frac{1}{12} \cdot [52 \cdot \ln(52) - 52] - \frac{1}{12} \cdot [7 \cdot \ln(7) - 7]$ ≈ 12,24