Lösungen zu den Übungen zu linearer Substitution

1. $\int_0^3 (5x-1)^3 \mathrm{dx} = \frac{1}{5} \cdot \left[\frac{1}{4} \cdot x^4\right]_{-1}^{14} = \frac{1}{20} \cdot 14^4 - \frac{1}{20} \cdot (-1)^4 = 1920{,}75$

2. $\int_2^4 (-7-3x)^5 dx = -\frac{1}{3} \cdot \left[\frac{1}{6} \cdot x^6\right]_{-13}^{-19} = -\frac{1}{18} \cdot (-19)^6 - (-\frac{1}{18} \cdot (-13)^6) = -2345504$

3. $\int_{-1}^3 2 \cdot \left(-x+\frac{2}{3}\right)^6 dx = (-1) \cdot \left[\frac{2}{7} \cdot x^7\right]_{\frac{5}{3}}^{-\frac{7}{3}} = -\frac{2}{7} \cdot (-\frac{7}{3})^7 - (-\frac{2}{7} \cdot \left(\frac{5}{3}\right)^7) = 117{,}796$

4. $\int_2^3 (ax-b)^7 dx = \frac{1}{a} \cdot \left[\frac{1}{8} \cdot x^8\right]_{2a-b}^{3a-b} = \frac{1}{8a} \cdot (3a-b)^8 - \frac{1}{8a} \cdot (2a-b)^8$

5. $\int_0^2 \sqrt{2x+8}\, dx = \frac{1}{2} \cdot \left[\frac{1}{1.5} \cdot x^{1,5}\right]_8^{12} = \frac{1}{3} \cdot 12^{1,5} - \frac{1}{3} \cdot 8^{1,5} \approx 6{,}3139$

6. $\int_{-4}^1 \sqrt{20-4x}\, dx = -\frac{1}{4} \cdot \left[\frac{1}{1.5} \cdot x^{1,5}\right]_{36}^{16} = -\frac{1}{6} \cdot (16)^{1,5} - (-\frac{1}{6} \cdot (36)^{1,5}) = 25{,}\overline{3}$

7. $\int_3^5 \sqrt[3]{5x-10}\, dx = \frac{1}{5} \cdot \left[\frac{3}{4} \cdot x^{\frac{4}{3}}\right]_5^{15} = \frac{3}{20} \cdot 15^{\frac{4}{3}} - \frac{3}{20} \cdot 5^{\frac{4}{3}} \approx 4{,}2665$

8. $\int_{-1}^3 \sqrt[4]{12+10x}\, \mathrm{dx} = \frac{1}{10} \cdot \left[\frac{4}{5} \cdot x^{\frac{5}{4}}\right]_2^{42} = \frac{2}{25} \cdot 42^{\frac{5}{4}} - \frac{2}{25} \cdot 2^{\frac{5}{4}} \approx 8{,}3634$

9. $\int_{-3}^3 e^{3x+6} dx = \frac{1}{3} \cdot [e^x]_{-3}^{15} = \frac{1}{3} \cdot e^{15} - \frac{1}{3} \cdot e^{-3} \approx 1089672$

10. $\int_{-2}^1 (-3) \cdot e^{-0,5x+2} dx = \frac{-3}{-0,5} \cdot [e^x]_3^{1,5} = 6 \cdot e^{1,5} - 6 \cdot e^3 \approx -93{,}623$

11. $\int_1^2 \frac{1}{e^{-2x-8}} dx = \int_1^2 e^{2x+8} dx = \frac{1}{2} \cdot [e^x]_{10}^{12} = \frac{1}{2} \cdot e^{12} - \frac{1}{2} \cdot e^{10} \approx 70364{,}2$

12. $\int_a^b 4 \cdot e^{kx} dx = \frac{4}{k} \cdot [e^x]_{ka}^{kb} = \frac{4}{k} \cdot e^{kb} - \frac{4}{k} \cdot e^{ka}$

13. $\int_0^{\pi} \sin(6x)\, dx = \frac{1}{6} \cdot [-\cos(x)]_0^{6\pi} = \frac{1}{6} \cdot (-\cos(6\pi)) - \frac{1}{6} \cdot (-\cos(0)) = 0$

14. $\int_{0,5}^{1,5} -6 \cdot \cos(-4x+1)\, dx = \frac{-6}{-4} \cdot [\sin(x)]_{-1}^{-5} = 1,5 \cdot \sin(-5) - 1,5 \cdot \sin(-1) \approx 2,7$

15. $\int_0^2 \sin\big(2 \cdot (4x-3)\big)\, dx = \int_0^2 \sin(8x-6)\, dx$

$= \frac{1}{8} \cdot [-\cos(x)]_{-6}^{10} = \frac{1}{8} \cdot (-\cos(10)) - \frac{1}{6} \cdot (-\cos(-6)) \approx 0,225$

16. $\int_{-2}^{5} \sin(ax^2+b)\, da = = \frac{1}{x^2} \cdot [-\cos(x)]_{-2x^2+b}^{5x^2+b}$

$= \frac{1}{x^2} \cdot (-\cos(5x^2+b)) - \frac{1}{x^2} \cdot (-\cos(-2x^2+b)) = -\frac{1}{x^2} \cdot \cos(5x^2+b) + \frac{1}{x^2} \cdot \cos(-2x^2+b)$