Lösungen zu den Übungen zur Fläche zwischen zwei Funktionen

1.a. $f(x) = -x^2 + 8x$ $g(x) = x^2$	$f(x) = g(x) \Leftrightarrow -x^2 + 8x = x^2 \Leftrightarrow -2x^2 + 8x = 0 \Leftrightarrow x = 0 \vee x = 4$ $f(1) = 7$ und $g(1) = 1$, d.h. $f(x) > g(x)$ für $0 \leq x \leq 4$ $A = \int_0^4 [f(x) - g(x)]dx = \int_0^4 (-2x^2 + 8x)dx$ $= \left[-\frac{2}{3}x^3 + 4x^2\right]_0^4 = \mathbf{\frac{64}{3}}$
b. $f(x) = x^2 + 2$ $g(x) = -x^3 + 3x^2 + 2$	$f(x) = g(x) \Leftrightarrow x^2 + 2 = -x^3 + 3x^2 + 2$ $\Leftrightarrow x^3 - 2x^2 = 0 \Leftrightarrow x = 0 \vee x = 2$ $A = \left\|\int_0^2 (x^3 - 2x^2)dx\right\| = \left\|\left[\frac{1}{4}x^4 - \frac{2}{3}x^3\right]_0^2\right\| = \left\|-\frac{4}{3}\right\| = \mathbf{\frac{4}{3}}$
c. $f(x) = -\frac{2}{3}x^3 + x^2$ $g(x) = -\frac{1}{3}x^2 + \frac{2}{3}x$	$f(x) = g(x) \Leftrightarrow -\frac{2}{3}x^3 + x^2 = -\frac{1}{3}x^2 + \frac{2}{3}x$ $\Leftrightarrow -\frac{2}{3}x^3 + \frac{4}{3}x^2 - \frac{2}{3}x = 0 \Leftrightarrow x = 0 \vee x = 1$ $A = \left\|\int_0^1 (-\frac{2}{3}x^3 + \frac{4}{3}x^2 - \frac{2}{3}x)\, dx\right\|$ $= \left[-\frac{1}{6}x^4 + \frac{4}{9}x^3 - \frac{1}{3}x^2\right]_0^1 = \left\|-\frac{1}{18}\right\| = \mathbf{\frac{1}{18}}$
d. $f(x) = x^3 - x$ $g(x) = -x^2 + 1$	$f(x) = g(x) \Leftrightarrow x^3 - x = -x^2 + 1 \Leftrightarrow x^3 + x^2 - x - 1 = 0$ $\Leftrightarrow x = -1 \vee x = 1$ $A = \left\|\int_{-1}^1 (x^3 + x^2 - x - 1)\, dx\right\|$ $= \left\|\left[\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{1}{2}x^2 - x\right]_{-1}^1\right\| = \left\|-\frac{4}{3}\right\| = \mathbf{\frac{4}{3}}$
e. $f(x) = -x^4 + 4x^2$ $g(x) = x^2 + 2x$	$f(x) = g(x) \Leftrightarrow -x^4 + 4x^2 = x^2 + 2x \Leftrightarrow -x^4 + 3x^2 - 2x = 0$ $\Leftrightarrow x = -2 \vee x = 0 \vee x = 1$ $A = \left\|\int_{-2}^0 (-x^4 + 3x^2 - 2x)\, dx\right\| + \left\|\int_0^1 (-x^4 + 3x^2 - 2x)\, dx\right\|$ $= \left\|\left[-\frac{1}{5}x^5 + \frac{3}{4}x^3 - x^2\right]_{-2}^0\right\| + \left\|\left[-\frac{1}{5}x^5 + \frac{3}{4}x^3 - x^2\right]_0^1\right\|$ $= \|5{,}6\| + \|-0{,}2\| = 5{,}6 + 0{,}2 =$ **5,8**

f. f(x) = $-x^3 + 4x$ g(x) = $-9x + 12$	f(x) = g(x) ⇔ $-x^3 + 4x = -9x + 12$ ⇔ $-x^3 + 13x - 12 = 0$ ⇔ x = −4 v x = 1 v x = 3 A = $\left\|\int_{-4}^{1}(-x^3 + 13x - 12)\,dx\right\| + \left\|\int_{1}^{3}(-x^3 + 13x - 12)\,dx\right\|$ = $\left\|\left[-\frac{1}{4}x^4 + 6{,}5x^2 - 12x\right]_{-4}^{1}\right\| + \left\|\left[-\frac{1}{4}x^4 + 6{,}5x^2 - 12x\right]_{1}^{3}\right\|$ = $\|-93{,}75\| + \|8\|$ = **101,75**
g. f(x) = $x^3 - x$ g(x) = $-x^3 + x^2 + 2x$	f(x) = g(x) ⇔ $x^3 - x = -x^3 + x^2 + 2x$ ⇔ $2x^3 - x^2 - 3x = 0$ x = −1 v x = 0 v x = 1,5 A = $\left\|\int_{-1}^{0}(2x^3 - x^2 - 3x)\,dx\right\| + \left\|\int_{0}^{1{,}5}(2x^3 - x^2 - 3x)\,dx\right\|$ = $\left\|\left[\frac{1}{2}x^4 - \frac{1}{3}x^3 - 1{,}5x^2\right]_{-1}^{0}\right\| + \left\|\left[\frac{1}{2}x^4 - \frac{1}{3}x^3 - 1{,}5x^2\right]_{0}^{1{,}5}\right\|$ = $\left\|\frac{2}{3}\right\| + \|-1{,}96875\|$ ≈ **2,63542**
h. f(x) = $x^4 + x^2$ g(x) = $4x^3 - 6x$	f(x) = g(x) ⇔ $x^4 + x^2 = 4x^3 - 6x$ ⇔ $x^4 - 4x^3 + x^2 + 6x = 0$ ⇔ x = −1 v x = 0 v x = 2 v x = 3 A = $\left\|\int_{-1}^{0}(x^4 - 4x^3 + x^2 + 6x)\,dx\right\|$ + $\left\|\int_{0}^{2}(x^4 - 4x^3 + x^2 + 6x)\,dx\right\|$ + $\left\|\int_{2}^{3}(x^4 - 4x^3 + x^2 + 6x)\,dx\right\|$ = $\left\|\left[\frac{1}{5}x^5 - x^4 + \frac{1}{3}x^3 + 3x^2\right]_{-1}^{0}\right\| + \left\|\left[\frac{1}{5}x^5 - x^4 + \frac{1}{3}x^3 + 3x^2\right]_{0}^{2}\right\|$ + $\left\|\left[\frac{1}{5}x^5 - x^4 + \frac{1}{3}x^3 + 3x^2\right]_{2}^{3}\right\|$ = $\left\|-\frac{22}{15}\right\| + \left\|\frac{76}{15}\right\| + \left\|-\frac{22}{15}\right\|$ = **8**