

Lösungen zu den Übungen zur Fläche zwischen zwei Funktionen

<p>1.a. $f(x) = -x^2 + 8x$ $g(x) = x^2$</p>	$f(x) = g(x) \Leftrightarrow -x^2 + 8x = x^2 \Leftrightarrow -2x^2 + 8x = 0 \Leftrightarrow x = 0 \vee x = 4$ $f(1) = 7$ und $g(1) = 1$, d.h. $f(x) > g(x)$ für $0 \leq x \leq 4$ $A = \int_0^4 [f(x) - g(x)] dx = \int_0^4 (-2x^2 + 8x) dx$ $= \left[-\frac{2}{3}x^3 + 4x^2 \right]_0^4 = \frac{32}{3}$
<p>b. $f(x) = x^2 + 2$ $g(x) = -x^3 + 3x^2 + 2$</p>	$f(x) = g(x) \Leftrightarrow x^2 + 2 = -x^3 + 3x^2 + 2$ $\Leftrightarrow x^3 - 2x^2 = 0 \Leftrightarrow x = 0 \vee x = 2$ $A = \left \int_0^2 (x^3 - 2x^2) dx \right = \left \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 \right]_0^2 \right = \left -\frac{4}{3} \right = \frac{4}{3}$
<p>c. $f(x) = -\frac{2}{3}x^3 + x^2$ $g(x) = -\frac{1}{3}x^2 + \frac{2}{3}x$</p>	$f(x) = g(x) \Leftrightarrow -\frac{2}{3}x^3 + x^2 = -\frac{1}{3}x^2 + \frac{2}{3}x$ $\Leftrightarrow -\frac{2}{3}x^3 + \frac{4}{3}x^2 - \frac{2}{3}x = 0 \Leftrightarrow x = 0 \vee x = 1$ $A = \left \int_0^1 \left(-\frac{2}{3}x^3 + \frac{4}{3}x^2 - \frac{2}{3}x \right) dx \right $ $= \left[-\frac{1}{6}x^4 + \frac{4}{9}x^3 - \frac{1}{3}x^2 \right]_0^1 = \left -\frac{1}{18} \right = \frac{1}{18}$
<p>d. $f(x) = x^3 - x$ $g(x) = -x^2 + 1$</p>	$f(x) = g(x) \Leftrightarrow x^3 - x = -x^2 + 1 \Leftrightarrow x^3 + x^2 - x - 1 = 0$ $\Leftrightarrow x = -1 \vee x = 1$ $A = \left \int_{-1}^1 (x^3 + x^2 - x - 1) dx \right $ $= \left \left[\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{1}{2}x^2 - x \right]_{-1}^1 \right = \left -\frac{4}{3} \right = \frac{4}{3}$
<p>e. $f(x) = -x^4 + 4x^2$ $g(x) = x^2 + 2x$</p>	$f(x) = g(x) \Leftrightarrow -x^4 + 4x^2 = x^2 + 2x \Leftrightarrow -x^4 + 3x^2 - 2x = 0$ $\Leftrightarrow x = -2 \vee x = 0 \vee x = 1$ $A = \left \int_{-2}^0 (-x^4 + 3x^2 - 2x) dx \right + \left \int_0^1 (-x^4 + 3x^2 - 2x) dx \right $ $= \left \left[-\frac{1}{5}x^5 + \frac{3}{4}x^3 - x^2 \right]_{-2}^0 \right + \left \left[-\frac{1}{5}x^5 + \frac{3}{4}x^3 - x^2 \right]_0^1 \right $ $= 5,6 + -0,2 = 5,6 + 0,2 = 5,8$

<p>f.</p> $f(x) = -x^3 + 4x$ $g(x) = -9x + 12$	$f(x) = g(x) \Leftrightarrow -x^3 + 4x = -9x + 12 \Leftrightarrow -x^3 + 13x - 12 = 0$ $\Leftrightarrow x = -4 \vee x = 1 \vee x = 3$ $A = \left \int_{-4}^1 (-x^3 + 13x - 12) dx \right + \left \int_1^3 (-x^3 + 13x - 12) dx \right $ $= \left \left[-\frac{1}{4}x^4 + 6,5x^2 - 12x \right]_{-4}^1 \right + \left \left[-\frac{1}{4}x^4 + 6,5x^2 - 12x \right]_1^3 \right $ $= -93,75 + 8 $ $= \mathbf{101,75}$
<p>g.</p> $f(x) = x^3 - x$ $g(x) = -x^3 + x^2 + 2x$	$f(x) = g(x) \Leftrightarrow x^3 - x = -x^3 + x^2 + 2x \Leftrightarrow 2x^3 - x^2 - 3x = 0$ $x = -1 \vee x = 0 \vee x = 1,5$ $A = \left \int_{-1}^0 (2x^3 - x^2 - 3x) dx \right + \left \int_0^{1,5} (2x^3 - x^2 - 3x) dx \right $ $= \left \left[\frac{1}{2}x^4 - \frac{1}{3}x^3 - 1,5x^2 \right]_{-1}^0 \right + \left \left[\frac{1}{2}x^4 - \frac{1}{3}x^3 - 1,5x^2 \right]_0^{1,5} \right $ $= \left \frac{2}{3} \right + -1,96875 $ $\approx \mathbf{2,63542}$
<p>h.</p> $f(x) = x^4 + x^2$ $g(x) = 4x^3 - 6x$	$f(x) = g(x) \Leftrightarrow x^4 + x^2 = 4x^3 - 6x \Leftrightarrow x^4 - 4x^3 + x^2 + 6x = 0$ $\Leftrightarrow x = -1 \vee x = 0 \vee x = 2 \vee x = 3$ $A = \left \int_{-1}^0 (x^4 - 4x^3 + x^2 + 6x) dx \right $ $+ \left \int_0^2 (x^4 - 4x^3 + x^2 + 6x) dx \right $ $+ \left \int_2^3 (x^4 - 4x^3 + x^2 + 6x) dx \right $ $= \left \left[\frac{1}{5}x^5 - x^4 + \frac{1}{3}x^3 + 3x^2 \right]_{-1}^0 \right + \left \left[\frac{1}{5}x^5 - x^4 + \frac{1}{3}x^3 + 3x^2 \right]_0^2 \right $ $+ \left \left[\frac{1}{5}x^5 - x^4 + \frac{1}{3}x^3 + 3x^2 \right]_2^3 \right $ $= \left -\frac{22}{15} \right + \left \frac{76}{15} \right + \left -\frac{22}{15} \right $ $= \mathbf{8}$