Lösungen zu den Übungen zu Ortskurven

Aufgabe	Rechnung	Lösung
1. Bestimmen Sie die Ortskurven der Extremstellen! a. $f_a(x) = 3x^2+2ax+9$	a. $f_a'(x) = 6x + 2a$ $f_a'(x) = 0 \Leftrightarrow 6x + 2a = 0 \Leftrightarrow 6x = -2a \Leftrightarrow x = -\frac{1}{3}a$ $f_a''(x) = 6 > 0 \Rightarrow$ Minimum $f_a(-\frac{1}{3}a) = 3\cdot(-\frac{1}{3}a)^2+2a\cdot(-\frac{1}{3}a)+9 = \frac{1}{3}a^2 - \frac{2}{3}a^2 + 9 = -\frac{1}{3}a^2 + 9$ ⇨ $T(-\frac{1}{3}a / -\frac{1}{3}a^2 + 9)$ $x = -\frac{1}{3}a \Leftrightarrow a = -3x$ $y = -\frac{1}{3}(-3x)^2 + 9 = -3x^2 + 9$	a. $f(x) = -3x^2 + 9$
b. $f_a(x) = 4x^4+2a^3x-6$	b. $f_a'(x) = 16x^3 + 2a^3$ $f_a'(x) = 0 \Leftrightarrow 16x^3 + 2a^3 = 0 \Leftrightarrow 16x^3 = -2a^3 \Leftrightarrow x^3 = -\frac{1}{8}a^3 \Leftrightarrow x = -\frac{1}{2}a$ $f_a''(x) = 48x^2$ und $f_a''(-\frac{1}{2}a) = 12a^2 > 0 \Rightarrow$ Maximum $f_a(-\frac{1}{2}a) = 4(-\frac{1}{2}a)^4+2a^3\cdot(-\frac{1}{2}a) - 6 = \frac{1}{4}a^4 - a^4 - 6 = -\frac{3}{4}a^4 - 6$ ⇨ $H(-\frac{1}{2}a / -\frac{3}{4}a^4 - 6)$ $x = -\frac{1}{2}a \Leftrightarrow a = -2x$ $y = -\frac{3}{4}(-2x)^4 - 6 = -12x^4 - 6$	b. $f(x) = -12x^4 - 6$

c. $f_a(x) = (a-x)\cdot e^{2x+1}$	c. $f_a'(x) = -1\cdot e^{2x+1} + 2\cdot(a-x)\cdot e^{2x+1} = (-1+2a-2x)\cdot e^{2x+1}$ $f_a'(x) = 0 \Leftrightarrow e^{2x+1} = 0$ (geht nicht) v $-1+2a-2x = 0$ $\Leftrightarrow 2x = -1+2a \Leftrightarrow x = a-0{,}5$ $f_a''(x) = -2\cdot e^{2x+1} + 2\cdot(-1+2a-2x)\cdot e^{2x+1} = (-4+4a-4x)\cdot e^{2x+1}$ $f_a''(a-0{,}5) = (-4+4a-4(a-0{,}5))\cdot e^{2(a-0{,}5)+1} = (-2)\cdot e^{2a} < 0 \Rightarrow$ Maximum $f_a(a-0{,}5) = (a-(a-0{,}5))\cdot e^{2a-1+1} = 0{,}5\cdot e^{2a}$ ⇨ $H(a-0{,}5/0{,}5\cdot e^{2a})$ $x = a-0{,}5 \Leftrightarrow a = x+0{,}5$ $y = 0{,}5\cdot e^{2x+1}$	c. $f(x) = 0{,}5\cdot e^{2x+1}$
d. $f_a(x) = (2x+a)\cdot e^{3x+1}$	d. $f_a'(x) = 2\cdot e^{3x+1} + 3\cdot(2x+a)\cdot e^{3x+1} = (2+6x+3a)\cdot e^{3x+1}$ $f_a'(x) = 0 \Leftrightarrow e^{3x+1} = 0$ (geht nicht) v $2+6x+3a = 0$ $\Leftrightarrow 6x = -3a-2 \Leftrightarrow x = -\frac{1}{2}a-\frac{1}{3}$ $f_a''(x) = 6\cdot e^{3x+1} + 3\cdot(2+6x+3a)\cdot e^{3x+1} = (12+18x+9a)\cdot e^{3x+1}$ $f_a''(-\frac{1}{2}a-\frac{1}{3}) = (12+18\cdot(-\frac{1}{2}a-\frac{1}{3})+9a)\cdot e^{-1,5a}$ $= (12-9a-6+9a)\cdot e^{-1,5a} = 6\cdot e^{-1,5a} > 0 \Rightarrow$ Minimum $f_a(-\frac{1}{2}a-\frac{1}{3}) = (2(-\frac{1}{2}a-\frac{1}{3})+a)\cdot e^{-1,5a} = (-a-\frac{2}{3}+a)\cdot e^{-1,5a} = -\frac{2}{3}\cdot e^{-1,5a}$ ⇨ $T(-\frac{1}{2}a-\frac{1}{3}/-\frac{2}{3}\cdot e^{-1,5a})$ $x = -\frac{1}{2}a-\frac{1}{3} \Leftrightarrow a = -2x-\frac{2}{3}$ $y = -\frac{2}{3}\cdot e^{3x+1}$	d. $f(x) = -\frac{2}{3}\cdot e^{3x+1}$

2. Bestimmen Sie die Ortskurve der Wendestellen! a. $f_a(x) = 2x^3 - 3ax^2 + 10$	a. $f_a'(x) = 6x^2 - 6ax$; $f_a''(x) = 12x - 6a$; $f_a'''(x) = 12$ $f_a''(x) = 0 \Leftrightarrow 12x - 6a = 0 \Leftrightarrow x = 0{,}5a$ $f_a'''(0{,}5a) = 12 > 0$ $f_a(0{,}5a) = 2(0{,}5a)^3 - 3a(0{,}5a)^2 + 10 = 0{,}25a^3 - 0{,}75a^3 + 10 = -0{,}5a^3 + 10$ $\Rightarrow$ WP$(0{,}5a / -0{,}5a^3 + 10)$ $x = 0{,}5a \Leftrightarrow a = 2x$; $y = -0{,}5 \cdot (2x)^3 + 10 = -4x^3 + 10$	a. $f(x) = -4x^3 + 10$
b. $f_a(x) = 10a - 2ae^x + e^{2x}$, $a > 0$	b. $f_a'(x) = -2ae^x + 2e^{2x}$; $f_a''(x) = -2ae^x + 4e^{2x}$; $f_a'''(x) = -2ae^x + 8e^{2x}$ $f_a''(x) = 0 \Leftrightarrow -2ae^x + 4e^{2x} = 0 \Leftrightarrow e^x \cdot (-2a + 4e^x) = 0$ $\Leftrightarrow e^x = 0$ (geht nicht) $\vee$ $2a = 4e^x \Leftrightarrow x = \ln(0{,}5a)$ $f_a'''(\ln(0{,}5a)) = -2a \cdot e^{\ln(0{,}5a)} + 8 \cdot e^{2\ln(0{,}5a)} = -2a \cdot (0{,}5a) + 8 \cdot e^{\ln(0{,}5a)^2}$ $= -2a \cdot (0{,}5a) + 8 \cdot (0{,}5a)^2 = a^2 > 0$ $f_a(\ln(0{,}5a)) = 10a - 2ae^{\ln(0{,}5a)} + e^{2\ln(0{,}5a)}$ $= 10a - 2a \cdot 0{,}5a + (0{,}5a)^2 = 10a - 0{,}75a^2$ $\Rightarrow$ WP$(\ln(0{,}5a) / 10a - 0{,}75a^2)$ $x = \ln(0{,}5a) \Leftrightarrow a = 2e^x$ $y = 10 \cdot 2e^x - 0{,}75 \cdot (2e^x)^2 = 20e^x - 3e^{2x}$	b. $f(x) = 20e^x - 3e^{2x}$