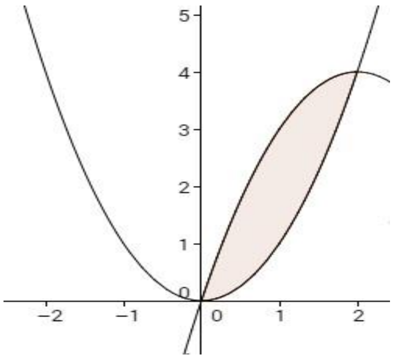
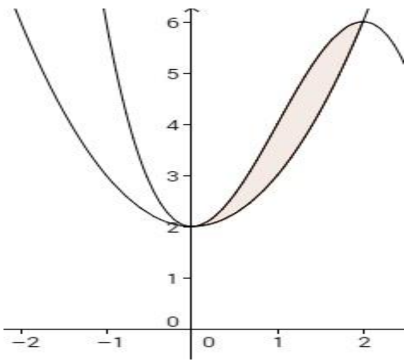
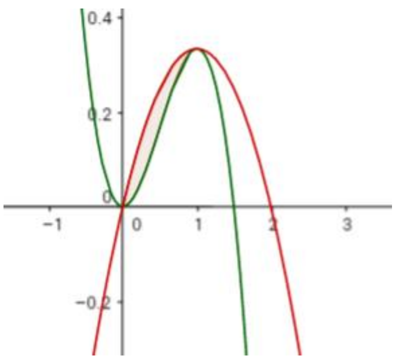
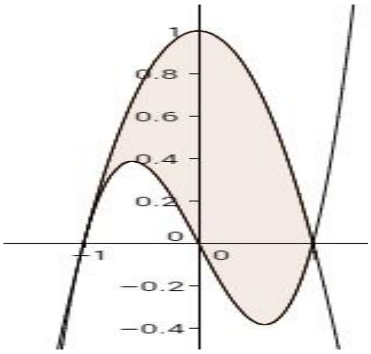
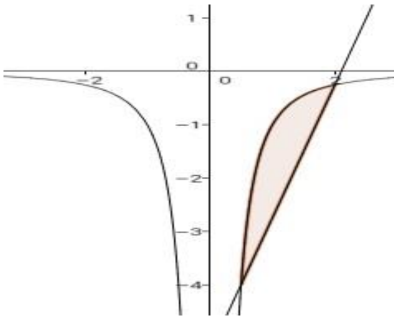
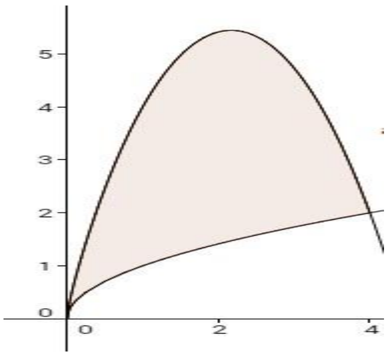
Lösungen zu den Übungen zur Fläche zwischen 2 Kurven mit 2 Schnittpunkten

$f(x) = -x^2 + 8x$ $g(x) = x^2$		Schnittpunkte: $S_1(0/0)$, $S_2(4/16)$ $f(1) = 7$ $g(1) = 1$, d.h. $f(x) > g(x)$ für $0 \leq x \leq 4$ $A = \int_0^4 f(x) - g(x)dx$ $= \int_0^4 -x^2 + 4x - (x^2)dx$ $= \int_0^4 (-2x^2 + 4x)dx$ $= \left[-\frac{2}{3}x^3 + 2x^2\right]_0^4$ $= \frac{32}{3} = 10,\overline{6}$
$f(x) = x^2 + 2$ $g(x) = -x^3 + 3x^2 + 2$		Schnittpunkte: $S_1(0/0)$, $S_2(2/4)$ $f(1) = 3$ $g(1) = 4$, d.h. $f(x) < g(x)$ für $0 \leq x \leq 2$ $A = \int_0^2 g(x) - f(x)dx$ $= \int_0^2 -x^3 + 3x^2 + 2 - (x^2 + 2)dx$ $= \int_0^2 (-x^3 + 2x^2)dx$ $= \left[-\frac{1}{4}x^4 + \frac{2}{3}x^3\right]_0^2$ $= \frac{4}{3} = 1,\overline{3}$
$f(x) = -\frac{2}{3}x^3 + x^2$ $g(x) = -\frac{1}{3}x^2 + \frac{2}{3}x$		Schnittpunkte: $S_1(0/0)$, $S_2(1/\frac{1}{3})$ $f(0,5) = 0,1\overline{6}$ $g(0,5) = 0,25$, d.h. $f(x) < g(x)$ für $0 \leq x \leq 2$ $A = \int_0^1 g(x) - f(x)dx$ $= \int_0^1 -\frac{1}{3}x^2 + \frac{2}{3}x - (-\frac{2}{3}x^3 + x^2)dx$ $= \int_0^1 (\frac{2}{3}x^3 - \frac{4}{3}x^2 + \frac{2}{3}x)\,dx$ $= \left[\frac{1}{6}x^4 - \frac{4}{9}x^3 + \frac{1}{3}x^2\right]_0^1$ $= \frac{1}{18} = 0,05\overline{6}$

$f(x) = x^3 - x$ $g(x) = -x^2 + 1$		Schnittpunkte: $S_1(-1/0)$, $S_2(1/0)$ $f(0) = 0$ $g(0) = 1$, d.h. $f(x) < g(x)$ für $-1 \leq x \leq 1$ $A = \int_{-1}^{1} g(x) - f(x)dx$ $= \int_{-1}^{1} -x^2 + 1 - (x^3 - x)dx$ $= \int_{-1}^{1}(-x^3 - x^2 + x + 1)\, dx$ $= \left[-\frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2 + x\right]_{-1}^{1}$ $= \frac{11}{12} - (-\frac{5}{12}) = \frac{16}{12} = \frac{4}{3}$
$f(x) = -\frac{1}{x^2}$ $g(x) = 2{,}5x - 5{,}25$ $x > 0$		Schnittpunkte: $S_1(0{,}5/-4)$, $S_2(2/-\frac{1}{4})$ $f(1) = -1$ $g(1) = -2{,}75$, d.h. $f(x) > g(x)$ für $0{,}5 \leq x \leq 2$ $A = \int_{0,5}^{2} f(x) - g(x)dx$ $= \int_{0,5}^{2}[-\frac{1}{x^2} - (2{,}5x - 5{,}25)]dx$ $= \int_{0,5}^{2}(-2{,}5x + 5{,}25 - \frac{1}{x^2})\, dx$ $= [-1{,}25x^2 + 5{,}25x + x^{-1}]_{0,5}^{2}$ $= 6 - (4{,}3125) = 1{,}6875$
$f(x) = -x^2 + 4x + \sqrt{x}$ $g(x) = \sqrt{x}$		Schnittpunkte: $S_1(0/0)$, $S_2(4/2)$ $f(1) = 4$ $g(1) = 1$, d.h. $f(x) > g(x)$ für $0 \leq x \leq 4$ $A = \int_{0}^{4} f(x) - g(x)dx$ $= \int_{0}^{4} -x^2 + 4x + \sqrt{x} - (\sqrt{x})dx$ $= \int_{0}^{4}(-x^2 + 4x)\, dx$ $= \left[-\frac{1}{3}x^3 + 2x^2\right]_{0}^{4}$ $= \frac{32}{3}$