Lösungen zu den Übungen zu linearen Gleichungssystemen mit 3 Unbekannten

Aufgabe	Rechenweg
1. $\left\vert \begin{matrix} x+y-z=9 \\ x+2y-4z=15 \\ x+3y-9z=23 \end{matrix} \right\vert$	$\left\vert \begin{matrix} x+y-z=9 \\ x+2y-4z=15 \\ x+3y-9z=23 \end{matrix} \right\vert \begin{matrix} I\cdot(-1) \\ II \\ III \end{matrix} \Leftrightarrow \left\vert \begin{matrix} -x-y+z=-9 \\ x+2y-4z=15 \\ x+3y-9z=23 \end{matrix} \right\vert \begin{matrix} I \\ II \\ III \end{matrix} \begin{matrix} \Rightarrow\ y-3z=6\ (I+II) \\ \Rightarrow\ 2y-8z=14\ (I+III) \end{matrix} \Leftrightarrow \left\vert \begin{matrix} x+y-z=9 \\ y-3z=6 \\ 2y-8z=14 \end{matrix} \right\vert$ $\left\vert \begin{matrix} x+y-z=9 \\ -2y+6z=-12 \\ 2y-8z=14 \end{matrix} \right\vert \Leftrightarrow \left\vert \begin{matrix} x+y-z=9 \\ -2y+6z=-12 \\ -2z=2 \end{matrix} \right\vert \Leftrightarrow \left\vert \begin{matrix} x+y-z=9 \\ -2y+6\cdot(-1)=-12 \\ z=-1 \end{matrix} \right\vert \Leftrightarrow \left\vert \begin{matrix} x+3-(-1)=9 \\ y=3 \\ z=-1 \end{matrix} \right\vert \Leftrightarrow \left\vert \begin{matrix} x=5 \\ y=3 \\ z=-1 \end{matrix} \right\vert$ IL= {(5/3/−1)}
2. $\left\vert \begin{matrix} 2x-y+4z=5 \\ 5x+2y-10z=7 \\ 12x-9y-8z=11 \end{matrix} \right\vert$	$\left\vert \begin{matrix} 2x-y+4z=5 \\ 5x+2y-10z=7 \\ 12x-9y-8z=11 \end{matrix} \right\vert \begin{matrix} I\cdot 2 \\ II \\ III \end{matrix} \Leftrightarrow \left\vert \begin{matrix} 4x-2y+8z=10 \\ 5x+2y-10z=7 \\ 12x-9y-8z=11 \end{matrix} \right\vert \begin{matrix} I \\ II \\ III \end{matrix} \Rightarrow 9x-2z=17\ (I+II) \Leftrightarrow \left\vert \begin{matrix} 2x-y+4z=5 \\ 5x+2y-10z=7 \\ 12x-9y-8z=11 \end{matrix} \right\vert \begin{matrix} I\cdot(-9) \\ II \\ III \end{matrix}$ $\left\vert \begin{matrix} -18x+9y-36z=-45 \\ 5x+2y-10z=7 \\ 12x-9y-8z=11 \end{matrix} \right\vert \begin{matrix} I \\ II \\ III \end{matrix} \Rightarrow -6x-44z=-34\ (I+III) \Leftrightarrow \left\vert \begin{matrix} 2x-y+4z=5 \\ 9x-2z=17 \\ -6x-44z=-34 \end{matrix} \right\vert \begin{matrix} I \\ II\cdot 2 \\ III\cdot 3 \end{matrix} \Leftrightarrow \left\vert \begin{matrix} 2x-y+4z=5 \\ 18x-4z=34 \\ -18x-132z=-102 \end{matrix} \right\vert$ $\left\vert \begin{matrix} 2x-y+4z=5 \\ 18x-4z=34 \\ -136z=-68 \end{matrix} \right\vert \begin{matrix} I \\ II \\ III:(-136) \end{matrix} \Leftrightarrow \left\vert \begin{matrix} 2x-y+4z=5 \\ 18x-4\cdot 0{,}5=34 \\ z=0{,}5 \end{matrix} \right\vert \Leftrightarrow \left\vert \begin{matrix} 2\cdot 2-y+4\cdot 0{,}5=5 \\ x=2 \\ z=0{,}5 \end{matrix} \right\vert \Leftrightarrow \left\vert \begin{matrix} y=1 \\ x=2 \\ z=0{,}5 \end{matrix} \right\vert$ IL= {(2/1/0,5)}
3. $\left\vert \begin{matrix} 3x-4y+2z=10 \\ 5x-3y+4z=3 \\ -2x+5y-3z=-7 \end{matrix} \right\vert$	$\left\vert \begin{matrix} 3x-4y+2z=10 \\ 5x-3y+4z=3 \\ -2x+5y-3z=-7 \end{matrix} \right\vert \begin{matrix} I\cdot(-2) \\ II \\ III \end{matrix} \Leftrightarrow \left\vert \begin{matrix} -6x+8y-4z=-20 \\ 5x-3y+4z=3 \\ -2x+5y-3z=-7 \end{matrix} \right\vert \begin{matrix} I \\ II \\ III \end{matrix} \Rightarrow -x+5y=-17\ (I+II) \Leftrightarrow \left\vert \begin{matrix} 3x-4y+2z=10 \\ 5x-3y+4z=3 \\ -2x+5y-3z=-7 \end{matrix} \right\vert \begin{matrix} I\cdot 1{,}5 \\ II \\ III \end{matrix}$ $\left\vert \begin{matrix} 4{,}5x-6y+3z=15 \\ 5x-3y+4z=3 \\ -2x+5y-3z=-7 \end{matrix} \right\vert \begin{matrix} I \\ II \\ III \end{matrix} \Rightarrow 2{,}5x-y=8\ (I+III) \left\vert \begin{matrix} 3x-4y+2z=10 \\ -x+5y=-17 \\ 2{,}5x-y=8 \end{matrix} \right\vert \begin{matrix} I \\ II \\ III\cdot 5 \end{matrix} \Leftrightarrow \left\vert \begin{matrix} 3x-4y+2z=10 \\ -x+5y=-17 \\ 12{,}5x-5y=40 \end{matrix} \right\vert \Leftrightarrow \left\vert \begin{matrix} 3x-4y+2z=10 \\ -x+5y=-17 \\ 11{,}5x=23 \end{matrix} \right\vert \begin{matrix} I \\ II \\ III:11{,}5 \end{matrix}$

	$\left\vert\begin{matrix}3x-4y+2z=10\\ -x+5y=-17\\ x=2\end{matrix}\right\vert \Leftrightarrow \left\vert\begin{matrix}3x-4y+2z=10\\ -2+5y=-17\\ x=2\end{matrix}\right\vert \Leftrightarrow \left\vert\begin{matrix}3\cdot 2-4\cdot(-3)+2z=10\\ y=-3\\ x=2\end{matrix}\right\vert \Leftrightarrow \left\vert\begin{matrix}z=-4\\ y=-3\\ x=2\end{matrix}\right\vert$ IL= {(2/−3/−4)}
4. $\left\vert\begin{matrix}4x+2y+2z=8\\ 3x-4y+3z=-2\\ x+3y+2z=4\end{matrix}\right\vert$*	$\left\vert\begin{matrix}4x+2y+2z=8\\ 3x-4y+3z=-2\\ x+3y+2z=4\end{matrix}\right\vert \begin{matrix}\cdot 3\\ \cdot(-4)\\ \cdot(-12)\end{matrix} \Leftrightarrow \left\vert\begin{matrix}12x+6y+6z=24\\ -12x+16y-12z=8\\ -12x-36y-24z=-48\end{matrix}\right\vert \Leftrightarrow \left\vert\begin{matrix}12x+6y+6z=24\\ 22y-6z=32\\ -30y-18z=-24\end{matrix}\right\vert \begin{matrix}I\\ \cdot(-3)\\ III\end{matrix} \Leftrightarrow \left\vert\begin{matrix}12x+6y+6z=24\\ -66y+18z=-96\\ -30y-18z=-24\end{matrix}\right\vert$ $\left\vert\begin{matrix}12x+6y+6z=24\\ -66y+18z=-96\\ -96y=-120\end{matrix}\right\vert \Leftrightarrow \left\vert\begin{matrix}12x+6y+6z=24\\ -66\cdot 1{,}25+18z=-96\\ y=\mathbf{1,25}\end{matrix}\right\vert \Leftrightarrow \left\vert\begin{matrix}12x+6y+6z=24\\ 18z=-96+82{,}5\\ y=1{,}25\end{matrix}\right\vert \Leftrightarrow \left\vert\begin{matrix}12x+6y+6z=24\\ z=-0{,}75\\ y=1{,}25\end{matrix}\right\vert$ $\left\vert\begin{matrix}12x+6\cdot 1{,}25+6\cdot(-0{,}75)=24\\ z=-0{,}75\\ y=1{,}25\end{matrix}\right\vert \Leftrightarrow \left\vert\begin{matrix}12x=21\\ z=0{,}75\\ y=1{,}25\end{matrix}\right\vert$ IL= {(1,75/1,25/−0,75)}
5. $\left\vert\begin{matrix}3x+6y-12z=26\\ x+5y+20z=-12\\ 5x+7y-50z=59\end{matrix}\right\vert$	$\left\vert\begin{matrix}3x+6y-12z=26\\ x+5y+20z=-12\\ 5x+7y-50z=59\end{matrix}\right\vert \begin{matrix}I\\ II\cdot(-3)\\ III\end{matrix} \Leftrightarrow \left\vert\begin{matrix}3x+6y-12z=26\\ -3x-15y-60z=36\\ 5x+7y-50z=59\end{matrix}\right\vert$ => −9y−72z = 62 (I + II) $\Leftrightarrow \left\vert\begin{matrix}3x+6y-12z=26\\ x+5y+20z=-12\\ 5x+7y-50z=59\end{matrix}\right\vert \begin{matrix}I\\ II\cdot(-5)\\ III\end{matrix}$ $\left\vert\begin{matrix}3x+6y-12z=26\\ -5x-25y-100z=60\\ 5x+7y-50z=59\end{matrix}\right\vert$ => $-18y-150z=119$ (II + III) $\Leftrightarrow \left\vert\begin{matrix}3x+6y-12z=26\\ -9y-72z=62\\ -18y-150z=119\end{matrix}\right\vert \begin{matrix}I\\ II\cdot(-2)\\ III\end{matrix} \Leftrightarrow \left\vert\begin{matrix}3x+6y-12z=26\\ 18y+144z=-124\\ -18y-150z=119\end{matrix}\right\vert$ $\left\vert\begin{matrix}3x+6y-12z=26\\ 18y+144z=-124\\ -6z=-5\end{matrix}\right\vert \Leftrightarrow \left\vert\begin{matrix}3x+6y-12z=26\\ -9y-72\cdot\frac{5}{6}=62\\ z=\frac{5}{6}\end{matrix}\right\vert \Leftrightarrow \left\vert\begin{matrix}3x+6y-12z=26\\ y=-\frac{122}{12}\\ z=\frac{5}{6}\end{matrix}\right\vert \Leftrightarrow \left\vert\begin{matrix}x=\frac{352}{9}\\ y=-\frac{122}{12}\\ z=\frac{5}{6}\end{matrix}\right\vert$ IL= $\{(\frac{352}{9}/-\frac{122}{12}/\frac{5}{6})\}$

6.

$$\left|\begin{array}{c} x+2y+3z=4 \\ 5x+6y+7z=8 \\ 9x+10y+11z=12 \end{array}\right|$$

$$\left|\begin{array}{c} x+2y+3z=4 \\ 5x+6y+7z=8 \\ 9x+10y+11z=12 \end{array}\right| \begin{array}{c} I \cdot(-5) \\ II \\ III \end{array} \Leftrightarrow \left|\begin{array}{c} -5x-10y-15z=-20 \\ 5x+6y+7z=8 \\ 9x+10y+11z=12 \end{array}\right| \Rightarrow -4y-8z=-12\ (I+II) \Leftrightarrow \left|\begin{array}{c} x+2y+3z=4 \\ 5x+6y+7z=8 \\ 9x+10y+11z=12 \end{array}\right| \begin{array}{c} I \cdot(-9) \\ II \\ III \end{array} \Leftrightarrow$$

$$\left|\begin{array}{c} -9x-18y-27z=-36 \\ 5x+6y+7z=8 \\ 9x+10y+11z=12 \end{array}\right| \Rightarrow -8y-16z=-24\ (I+III) \Leftrightarrow \left|\begin{array}{c} x+2y-2z=-1 \\ -4y-8z=-12 \\ -8y-16z=-24 \end{array}\right| \begin{array}{c} I \\ II \cdot(-2) \\ III \end{array} \Leftrightarrow \left|\begin{array}{c} x+2y-2z=-1 \\ 8y+16z=24 \\ -8y-16z=-24 \end{array}\right| \Leftrightarrow \left|\begin{array}{c} x+2y-2z=-1 \\ 8y-28z=-22 \\ \mathbf{0=0} \end{array}\right|$$

$\infty - viele\ Lösungen$

7.

$$\left|\begin{array}{c} 2x+4y+4z=-4 \\ 2x+3y+z=1 \\ 3x+5y+3z=-1 \end{array}\right|$$

$$\left|\begin{array}{c} 2x+4y+4z=-4 \\ 2x+3y+z=1 \\ 3x+5y+3z=-1 \end{array}\right| \begin{array}{c} I \\ II \cdot(-4) \\ III \end{array} \Leftrightarrow \left|\begin{array}{c} 2x+4y+4z=-4 \\ -8x-12y-4z=-4 \\ 3x+5y+3z=-1 \end{array}\right| \Rightarrow -6x-8y=-8\ (I+II) \Leftrightarrow \left|\begin{array}{c} 2x+4y+4z=-4 \\ 2x+3y+z=1 \\ 3x+5y+3z=-1 \end{array}\right| \begin{array}{c} I \\ II \cdot(-3) \\ III \end{array} \Leftrightarrow \left|\begin{array}{c} 2x+4y+4z=-4 \\ -6x-9y-3z=-3 \\ 3x+5y+3z=-1 \end{array}\right|$$

$$\Rightarrow -3x-4y=-4\ (II+III) \Leftrightarrow \left|\begin{array}{c} 2x+4y+4z=-4 \\ -6x-8y=-8 \\ -3x-4y=-4 \end{array}\right| \begin{array}{c} I \\ II \\ III \cdot(-2) \end{array} \Leftrightarrow \left|\begin{array}{c} 2x+4y+4z=-4 \\ -6x-8y=-8 \\ 6x+8y=8 \end{array}\right| \Leftrightarrow \left|\begin{array}{c} 2x+4y+4z=-4 \\ -6x-8y=-8 \\ \mathbf{0=0} \end{array}\right|$$

$\infty - viele\ Lösungen$

8.

$$\left|\begin{array}{c} 4x+2y+2z=8 \\ 3x+9y+6z=-2 \\ x+3y+2z=4 \end{array}\right| **$$

$$\left|\begin{array}{c} 4x+2y+2z=8 \\ 3x+9y+6z=-2 \\ x+3y+2z=4 \end{array}\right| \begin{array}{c} \cdot(-3) \\ II \\ \cdot(-3) \end{array} \Leftrightarrow \left|\begin{array}{c} -12x-6y-6z=-24 \\ 3x+9y+6z=-2 \\ -3x-9y-6z=-12 \end{array}\right| \Leftrightarrow \left|\begin{array}{c} -9x+3y=-26 \\ 3x+9y+6z=-2 \\ \mathbf{0=-14} \end{array}\right|$$

Die III. Zeile beinhaltet einen Widerspruch.

keine Lösung