Lösungen zu Aufgaben zu Maximum und Minimum

Berechnen Sie die lokalen Extrema!

Aufgabe	Rechenweg	Lösung
1. $f(x) = x^2 - 8x + 19$	$f'(x) = 2x - 8$ $f'(x) = 0 \Leftrightarrow x = 4$ $f''(x) = 2$ $f''(4) = 2 > 0 \Rightarrow$ Minimum $f(4) = 3$	Minimum (4/3)
2. $f(x) = \frac{1}{3}x^3 + 4x^2 - 20x + 4$	$f'(x) = x^2 + 8x - 20$ $f'(x) = 0 \Leftrightarrow x = -10 \vee x = 2$ (TR oder p-q-Formel) $f''(x) = 2x + 8$ $f''(-10) = -12 < 0 \Rightarrow$ Maximum $f(-10) = \frac{812}{3} = 270,\overline{6}$ $f''(2) = 12 > 0 \Rightarrow$ Minimum $f(2) = -\frac{52}{3} = -17,\overline{3}$	Minimum $(2/-17,\overline{3})$ Maximum $(-10/270,\overline{6})$
3. $f(x) = 0{,}5x^4 + \frac{14}{3}x^3 + 7x^2 - 30x + 10$	$f'(x) = 2x^3 + 14x^2 + 14x - 30$ $f'(x) = 0 \Leftrightarrow x = -5 \vee x = -3 \vee x = 1$ (TR oder Polynomdivision) $f''(x) = 6x^2 + 28x + 14$ $f''(-5) = 24 > 0 \Rightarrow$ Minimum $f(-5) = 64,1\overline{6}$ $f''(-3) = -16 < 0 \Rightarrow$ Maximum $f(-3) = 77,5$ $f''(1) = 48 > 0 \Rightarrow$ Minimum $f(1) = -7,8\overline{3}$	Minimum $(-5/64,1\overline{6})$ Minimum $(1/-7,8\overline{3})$ Maximum $(-3/77,5)$

4. $f(x) = x^3 + 12x^2 - 27x + 30$	$f'(x) = 3x^2 + 24x - 27$ $f'(x) = 0 \Leftrightarrow 3x^2 + 24x - 27 = 0 \Leftrightarrow x = -9 \vee x = 1$ $f''(x) = 6x + 24$ $f''(-9) = -30 < 0 \Rightarrow$ Maximum $f(-9) = 504$ $f''(1) = 30 > 0 \Rightarrow$ Minimum $f(1) = 16$	Maximum (–9/516) Minimum (1/16)
5. $f(x) = \frac{4}{3}x^3 - 4x^2 - 32x + 2$	$f'(x) = 4x^2 - 8x - 32$ $f'(x) = 0 \Leftrightarrow 4x^2 - 8x - 32 = 0 \Leftrightarrow x = -2 \vee x = 4$ $f''(x) = 8x - 8$ $f''(-2) = -24 < 0 \Rightarrow$ Maximum $f''(4) = 24 > 0 \Rightarrow$ Minimum $f(-2) = \frac{118}{3} = 39,\overline{3}$ $f(4) = -\frac{314}{3} = -104,\overline{6}$	Minimum $(4/-104,\overline{6})$ Maximum $(-2/39,\overline{3})$
6. $f(x) = 4x^5 + 20x$	$f'(x) = 20x^4 + 20$ $f'(x) = 0 \Leftrightarrow 20x^4 + 20 = 0 \Leftrightarrow 20x^4 = -20 \Leftrightarrow x^4 = -1$ keine Lösung	kein Maximum, kein Minimum