

Lösungen zu den Übungen zur Substitution: Sonderfälle

Aufgabe	Rechenweg	Lösung
a. $x^6 - 19x^3 - 216 = 0$	$x^3 = z \Rightarrow z^2 - 19z - 216 = 0$ $\Leftrightarrow z_{1,2} = -\frac{-19}{2} \pm \sqrt{\left(\frac{-19}{2}\right)^2 - (-216)}$ $z_{1,2} = 9,5 \pm 17,5$ $\Leftrightarrow z_1 = 27 \vee z_2 = -10$ $\Leftrightarrow x^3 = 27 \vee x^3 = -10$ $\Leftrightarrow x = 3 \vee x = -\sqrt[3]{10} (\approx -2,15)$	$x = 3$ $x = -\sqrt[3]{10}$
b. $2x^6 - 18x^3 + 16 = 0$	$x^3 = z \Rightarrow 2z^2 - 18z + 16 = 0 \quad /:2$ $\Leftrightarrow z^2 - 9z + 8 = 0$ $\Leftrightarrow z_{1,2} = -\frac{-9}{2} \pm \sqrt{\left(\frac{9}{2}\right)^2 - 8}$ $z_{1,2} = 4,5 \pm 3,5$ $\Leftrightarrow z_1 = 1 \vee z_2 = 8$ $\Leftrightarrow x^3 = 1 \vee x^3 = 8$ $\Leftrightarrow x = 1 \vee x = 2$	$x = 1$ $x = 2$
c. $x^8 + \frac{63}{16}x^4 - \frac{1}{4} = 0$	$x^4 = z \Rightarrow z^2 + \frac{63}{16}z - \frac{1}{4} = 0$ $z_{1,2} = -\frac{63}{32} \pm \sqrt{\left(\frac{63}{32}\right)^2 - \left(-\frac{1}{4}\right)}$ $z_{1,2} = -\frac{63}{32} \pm \frac{65}{32}$ $\Leftrightarrow z_1 = -\frac{128}{32} = -4 \vee z_2 = \frac{2}{32} = \frac{1}{16}$ $\Leftrightarrow x^4 = -4 \text{ (geht nicht)} \vee x^4 = \frac{1}{16}$ $\Leftrightarrow x = \frac{1}{2} \vee x = -\frac{1}{2}$	$x = \frac{1}{2}$ $x = -\frac{1}{2}$

<p>d. $4x^5 + 16x^3 - 62x = 0$</p>	$x \cdot (4x^4 + 16x^2 - 128) = 0$ $\Leftrightarrow x = 0 \quad \vee \quad 4x^4 + 16x^2 - 128 = 0 / :4$ $\Leftrightarrow x = 0 \quad \vee \quad x^4 + 4x^2 - 128 = 0 \quad (z = x^2)$ $\Leftrightarrow x = 0 \quad \vee \quad z^2 + 4z - 32 = 0$ $\Leftrightarrow x = 0 \quad \vee \quad z_{1,2} = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - (-32)}$ $\Leftrightarrow x = 0 \quad \vee \quad z_{1,2} = -2 \pm 6$ $\Leftrightarrow x = 0 \quad \vee \quad z = 4 \quad \vee \quad z = -8$ $\Leftrightarrow x = 0 \quad \vee \quad x^2 = 4 \quad \vee \quad x^2 = -8 \text{ (geht nicht)}$ $\Leftrightarrow x = 0 \quad \vee \quad x = 2 \quad \vee \quad x = -2$	$x = 0$ $x = 2$ $x = -2$
<p>e. $-x^{10} + 22x^5 + 320 = 0$</p>	$z = x^5 \Rightarrow -z^2 + 22z + 320 = 0 / \cdot (-1)$ $\Leftrightarrow z^2 - 22z - 320 = 0$ $\Leftrightarrow z_{1,2} = -\frac{-22}{2} \pm \sqrt{\left(\frac{22}{2}\right)^2 - (-320)}$ $\Leftrightarrow z_{1,2} = 11 \pm 21$ $\Leftrightarrow z_1 = 32 \quad \vee \quad z_2 = -10$ $\Leftrightarrow x^5 = 32 \quad \vee \quad x^5 = -10$ $\Leftrightarrow x = \sqrt[5]{32} = 2 \quad \vee \quad x = -\sqrt[5]{10} (\approx -1,58)$	$x = 2$ $x = -\sqrt[5]{10} (\approx -1,58)$
<p>f. $3x^6 - 6x^4 - 189x^2 = 0$</p>	$x^2 \cdot (3x^4 - 6x^2 - 189) = 0$ $\Leftrightarrow x^2 = 0 \quad \vee \quad 3x^4 - 6x^2 - 189 = 0 / :3$ $\Leftrightarrow x = 0 \quad \vee \quad x^4 - 2x^2 - 63 = 0 \quad (z = x^2)$ $\Leftrightarrow x = 0 \quad \vee \quad z^2 - 2z - 63 = 0$ $\Leftrightarrow x = 0 \quad \vee \quad z_{1,2} = -\frac{-2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - (-63)}$ $\Leftrightarrow x = 0 \quad \vee \quad z_{1,2} = 1 \pm 8$ $\Leftrightarrow x = 0 \quad \vee \quad z_1 = 9 \quad \vee \quad z_2 = -7$ $\Leftrightarrow x = 0 \quad \vee \quad x^2 = 9 \quad \vee \quad x^2 = -7 \text{ (geht nicht)}$ $\Leftrightarrow x = 0 \quad \vee \quad x = 3 \quad \vee \quad x = -3$	$x = 0$ $x = 3$ $x = -3$

<p>g. $(x^2 - 9) \cdot (2x^4 - 28x^2 - 64) = 0$</p>	$x^2 - 9 = 0 \quad \vee \quad 2x^4 - 28x^2 - 64 = 0 \quad /:2$ $\Leftrightarrow x^2 = 9 \quad \vee \quad x^4 - 14x^2 - 32 = 0 \quad (z = x^2)$ $\Leftrightarrow x = 3 \vee x = -3 \quad \vee \quad z^2 - 14z - 32 = 0$ $\Leftrightarrow x = 3 \vee x = -3 \quad \vee \quad z_{1,2} = -\frac{-14}{2} \pm \sqrt{\left(\frac{14}{2}\right)^2 - (-32)}$ $\Leftrightarrow x = 3 \vee x = -3 \quad \vee \quad z_{1,2} = 7 \pm 9$ $\Leftrightarrow x = 3 \vee x = -3 \quad \vee \quad x^2 = 16 \vee x^2 = -8 \text{ (geht nicht)}$ $\Leftrightarrow x = 3 \vee x = -3 \quad \vee \quad x = 4 \vee x = -4$	$x = 3$ $x = -3$ $x = 4$ $x = -4$
<p>h. $(x + 3) \cdot (x^5 - 23x^3 - 50x) = 0$</p>	$(x + 3) = 0 \quad \vee \quad (x^5 - 23x^3 - 50x) = 0$ $\Leftrightarrow x = -3 \quad \vee \quad x \cdot (x^4 - 23x^2 - 50) = 0$ $\Leftrightarrow x = -3 \quad \vee \quad x = 0 \quad \vee \quad x^4 - 23x^2 - 50 = 0 \quad (z = x^2)$ $\Leftrightarrow x = -3 \quad \vee \quad x = 0 \quad \vee \quad z^2 - 23z - 50 = 0$ $\Leftrightarrow x = -3 \quad \vee \quad x = 0 \quad \vee \quad z_{1,2} = -\frac{-23}{2} \pm \sqrt{\left(\frac{23}{2}\right)^2 - (-50)}$ $\Leftrightarrow x = -3 \quad \vee \quad x = 0 \quad \vee \quad z_{1,2} = 11,5 \pm 13,5$ $\Leftrightarrow x = -3 \quad \vee \quad x = 0 \quad \vee \quad x^2 = 25 \vee x^2 = -2 \text{ (geht nicht)}$ $\Leftrightarrow x = -3 \quad \vee \quad x = 0 \quad \vee \quad x = 5 \quad \vee \quad x = -5$	$x = -3$ $x = 0$ $x = 5$ $x = -5$