

Lösung zu den Übungen zur Substitution I

Aufgabe	Rechenweg	Ergebnis
a. $x^4 - 8x^2 + 7 = 0$	$z^2 - 8z + 7 = 0 \Leftrightarrow z_{1,2} = -\frac{-8}{2} \pm \sqrt{\left(\frac{-8}{2}\right)^2 - 7} = 4 \pm 3$ $\Leftrightarrow z_1 = 1 \vee z_2 = 7 \quad \text{Rücksubstitution}$ $\Leftrightarrow x^2 = 1 \vee x^2 = 7$ $\Leftrightarrow x = 1 \vee x = -1 \vee x = \sqrt{7} \vee x = -\sqrt{7}$	$x = 1 \vee x = -1 \vee x = \sqrt{7}$ $\vee x = -\sqrt{7}$
b. $x^4 + 2x^2 - 8 = 0$	$z^2 + 2z - 8 = 0 \Leftrightarrow z_{1,2} = -\frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - (-8)} = -1 \pm 3$ $\Leftrightarrow z_1 = 2 \vee z_2 = -4$ $\Leftrightarrow x^2 = 2 \vee x^2 = -4 \text{ (geht nicht)}$ $\Leftrightarrow x = \sqrt{2} \vee x = -\sqrt{2}$	$x = \sqrt{2} \vee x = -\sqrt{2}$
c. $x^4 + 9x^2 + 20 = 0$	$z^2 + 9z + 20 = 0 \Leftrightarrow z_{1,2} = -\frac{9}{2} \pm \sqrt{\left(\frac{9}{2}\right)^2 - 20} = -4,5 \pm 0,5$ $\Leftrightarrow z_1 = -4 \vee z_2 = -5 \Leftrightarrow x^2 = -4 \vee x^2 = -5 \text{ (geht nicht)}$ <p>keine Lösung</p>	keine Lösung
d. $x^4 - 6x^2 + 16 = 8$	$z^2 - 6z + 8 = 0 \Leftrightarrow z_{1,2} = -\frac{-6}{2} \pm \sqrt{\left(\frac{-6}{2}\right)^2 - 8} = 3 \pm 1$ $\Leftrightarrow z_1 = 4 \vee z_2 = 2$ $\Leftrightarrow x^2 = 4 \vee x^2 = 2$ $\Leftrightarrow x = 2 \vee x = -2 \vee x = \sqrt{2} \vee x = -\sqrt{2}$	$x = 2 \vee x = -2 \vee x = \sqrt{2}$ $\vee x = -\sqrt{2}$

<p>e. $x^4 - 2x^2 = 24$</p>	$z^2 - 2z - 24 = 0 \Leftrightarrow z_{1,2} = -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 - (-24)} = 1 \pm 5$ <p> $\Leftrightarrow z_1 = 6 \vee z_2 = -4$ $\Leftrightarrow x^2 = 6 \vee x^2 = -4$ (geht nicht) $\Leftrightarrow x = \sqrt{6} \vee x = -\sqrt{6}$ </p>	$x = \sqrt{6} \vee x = -\sqrt{6}$
<p>f. $x^4 - 10x^2 + 30 = 0$</p>	$z^2 - 10z + 30 = 0 \Leftrightarrow z_{1,2} = -\frac{-10}{2} \pm \sqrt{\left(\frac{-10}{2}\right)^2 - 30} = 5 \pm \sqrt{-5}$ <p>keine Lösung, da die Wurzel aus einer negativen Zahl nicht gezogen werden kann</p>	keine Lösung
<p>g. $x^4 - 45x^2 + 324 = 0$</p>	$z^2 - 45z + 324 = 0 \Leftrightarrow z_{1,2} = -\frac{-45}{2} \pm \sqrt{\left(\frac{-45}{2}\right)^2 - 324} = 22,5 \pm 13,5$ <p> $\Leftrightarrow z_1 = 36 \vee z_2 = 9$ $\Leftrightarrow x^2 = 36 \vee x^2 = 9$ $\Leftrightarrow x = 6 \vee x = -6 \vee x = 3 \vee x = -3$ </p>	$x = 6 \vee x = -6 \vee x = 3 \vee x = -3$
<p>h. $x^4 + 4x^2 = -3$</p>	$z^2 + 4z + 3 = 0 \Leftrightarrow z_{1,2} = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 3} = -2 \pm 1$ <p> $\Leftrightarrow z_1 = -1 \vee z_2 = -3$ $\Leftrightarrow x^2 = -1 \vee x^2 = -3$ \Leftrightarrow keine Lösung </p>	keine Lösung

2. Löse die Gleichung, indem du zuerst den Faktor vor x^4 eliminiertest!

<p>a. $2x^4 + 8x^2 - 42 = 0$</p>	<p>$2x^4 + 8x^2 - 42 = 0 / :2 \Leftrightarrow x^4 + 4x^2 - 21 = 0$</p> <p>$z^2 + 4z - 21 = 0 \Leftrightarrow z_{1,2} = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - (-21)} = -2 \pm 5$</p> <p>$\Leftrightarrow z_1 = 3 \vee z_2 = -7 \Leftrightarrow x^2 = 3 \vee x^2 = -7$ (geht nicht) $\Leftrightarrow x = \sqrt{3} \vee x = -\sqrt{3}$</p>	<p>$x = \sqrt{3} \vee x = -\sqrt{3}$</p>
<p>b. $4x^4 - 16x^2 - 48 = 0$</p>	<p>$4x^4 - 16x^2 - 48 = 0 / :4 \Leftrightarrow x^4 - 4x^2 - 12 = 0$</p> <p>$z^2 - 4z - 12 = 0 \Leftrightarrow z_{1,2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{-4}{2}\right)^2 - (-12)} = 2 \pm 4$</p> <p>$\Leftrightarrow z_1 = 6 \vee z_2 = -2 \Leftrightarrow x^2 = 6 \vee x^2 = -2$ (geht nicht) $\Leftrightarrow x = \sqrt{6} \vee x = -\sqrt{6}$</p>	<p>$x = \sqrt{6} \vee x = -\sqrt{6}$</p>
<p>c. $-6x^4 - 12x^2 + 48 = 0$</p>	<p>$-6x^4 - 12x^2 + 48 = 0 / :(-6) \Leftrightarrow x^4 + 2x^2 - 8 = 0$</p> <p>$z^2 + 2z - 8 = 0 \Leftrightarrow z_{1,2} = -\frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - (-8)} = -1 \pm 3$</p> <p>$\Leftrightarrow z_1 = 2 \vee z_2 = -4 \Leftrightarrow x^2 = 2 \vee x^2 = -4$ (geht nicht) $\Leftrightarrow x = \sqrt{2} \vee x = -\sqrt{2}$</p>	<p>$x = \sqrt{2} \vee x = -\sqrt{2}$</p>
<p>d. $7x^4 + 21x^2 = -14$</p>	<p>$7x^4 + 21x^2 + 14 = 0 / :7 \Leftrightarrow x^4 + 3x^2 + 2 = 0$</p> <p>$z^2 + 3z + 2 = 0 \Leftrightarrow z_{1,2} = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - 2} = -1,5 \pm 0,5$</p> <p>$\Leftrightarrow z_1 = -1 \vee z_2 = -2 \Leftrightarrow x^2 = -1 \vee x^2 = -2 \Leftrightarrow$ keine Lösung</p>	<p>keine Lösung</p>
<p>e. $2x^4 - \frac{11}{6}x^2 + \frac{1}{3} = 0$</p>	<p>$2x^4 - \frac{11}{6}x^2 + \frac{1}{3} = 0 / :2 \Leftrightarrow x^4 - \frac{11}{12}x^2 + \frac{1}{6} = 0$</p> <p>$\Leftrightarrow z^2 - \frac{11}{12}z + \frac{1}{6} = 0 \Leftrightarrow z_{1,2} = -\frac{-11}{2} \pm \sqrt{\left(\frac{-11}{2}\right)^2 - \frac{1}{6}} = \frac{11}{24} \pm \frac{5}{24}$</p> <p>$\Leftrightarrow z_1 = \frac{16}{24} = \frac{2}{3} \vee z_2 = \frac{6}{24} = \frac{1}{4} \Leftrightarrow x^2 = \frac{2}{3} \vee x^2 = \frac{1}{4}$</p> <p>$\Leftrightarrow x = \sqrt{\frac{2}{3}} \vee x = -\sqrt{\frac{2}{3}} \vee x = \frac{1}{2} \vee x = -\frac{1}{2}$</p>	<p>$x = \sqrt{\frac{2}{3}} \vee x = -\sqrt{\frac{2}{3}}$ $\vee x = \frac{1}{2} \vee x = -\frac{1}{2}$</p>