

# Lösungen zu den zusammenfassenden Aufgaben Integralrechnung

1. Finden Sie die zugehörige Stammfunktion!

$f(x) = 2x^2 + 5x$	$F(x) = \frac{2}{3}x^3 + \frac{5}{2}x^2$
$f(x) = 8x^3 - 5x^2 + 3x - 6$	$F(x) = 2x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 - 6x$
$f(x) = \frac{2}{3}x^6 - x^5 + \frac{5}{6}x^3 - \frac{9}{16}x^2$	$F(x) = \frac{2}{21}x^7 - \frac{1}{6}x^6 + \frac{5}{24}x^4 - \frac{3}{16}x^3$
$f(x) = 3$	$F(x) = 3x$
$f(x) = 2x^4 \cdot (x^3 + 4)$ $= 2x^7 + 8x^4$	$F(x) = \frac{1}{4}x^8 + \frac{8}{5}x^5$
$f(x) = 4 \cdot e^{3x+4}$	$F(x) = \frac{4}{3} \cdot e^{3x+4}$
$f(x) = e^{-4x+2} + 8 \cdot e^{-6x+1}$	$F(x) = -\frac{1}{4} \cdot e^{-4x+2} - \frac{4}{3} \cdot e^{-6x+1}$
$f(x) = 1 - 8 \cdot e^{-5x+5}$	$F(x) = x + \frac{8}{5} \cdot e^{-5x+5}$
$f(x) = (4x - 3)^5$	$F(x) = \frac{1}{24} \cdot (4x - 3)^6$

2. Berechnen Sie die Fläche zwischen  $f(x)$  und der  $x$ -Achse!

$f(x) = 9 - x^2$	$9 - x^2 = 0 \Leftrightarrow x = -3 \vee x = 3$ <span style="float: right;"><math>(f(0) = 9, \text{ also } f(x) &gt; 0 \text{ in } [-3;3])</math></span> $A = \int_{-3}^3 (9 - x^2) dx = \left[ 9x - \frac{1}{3}x^3 \right]_{-3}^3 = 18 - (-18) = \mathbf{36}$
$f(x) = x^2 - 5x - 24$	$x^2 - 5x - 24 = 0 \Leftrightarrow x = -3 \vee x = 8$ <span style="float: right;"><math>(f(0) = -24, \text{ also } f(x) &lt; 0 \text{ in } [-3;8])</math></span> $A = \left  \int_{-3}^8 (x^2 - 5x - 24) dx \right  = \left  \left[ \frac{1}{3}x^3 - 2,5x^2 - 24x \right]_{-3}^8 \right $ $=  -181, \bar{3} - 40,5  =  -221,8\bar{3}  = \mathbf{221,8\bar{3}}$
$f(x) = x^3 + 3x^2 - 10x - 24$	$x^3 + 3x^2 - 10x - 24 = 0 \Leftrightarrow x = -4 \vee x = -2 \vee x = 3$ $A = \left  \int_{-4}^{-2} (x^3 + 3x^2 - 10x - 24) dx \right  + \left  \int_{-2}^3 (x^3 + 3x^2 - 10x - 24) dx \right $ $= \left  \left[ \frac{1}{4}x^4 + x^3 - 5x^2 - 24x \right]_{-4}^{-2} \right  + \left  \left[ \frac{1}{4}x^4 + x^3 - 5x^2 - 24x \right]_{-2}^3 \right $ $= 8 +  -93,75  = \mathbf{101,75}$

### 3. Berechnen Sie die Fläche zwischen den beiden Funktionen!

$f(x) = -8x + 12$ $g(x) = 4x^2$	$-8x + 12 = 4x^2 \Leftrightarrow 4x^2 + 8x - 12 = 0 \Leftrightarrow x = -3 \vee x = 1$ $(f(0) = 12, g(0) = 0 \Rightarrow f(x) > g(x) \text{ in } [-3;1])$ $A = \int_{-3}^1 (-4x^2 - 8x + 12) dx = \left[ -\frac{4}{3}x^3 - 4x^2 + 12x \right]_{-3}^1$ $= \frac{20}{3} - (-36) = \frac{128}{3} = \mathbf{42, \bar{6}}$
$f(x) = x^2 - x^4$ $g(x) = x^2 - 1$	$x^2 - x^4 = x^2 - 1 \Leftrightarrow x^4 = 1 \Leftrightarrow x = -1 \vee x = 1$ $(f(0) = 0, g(0) = -1 \Rightarrow f(x) > g(x) \text{ in } [-1;1])$ $A = \int_{-1}^1 (-x^4 + 1) dx = \left[ -\frac{1}{5}x^5 + x \right]_{-1}^1 = \frac{4}{5} - \left(-\frac{4}{5}\right)$ $= \frac{8}{5} = \mathbf{1,6}$

### 4. Berechnen Sie den Mittelwert der Funktion im angegebenen Intervall !!

$f(x) = -x^2 + 16$ $I = [1;3]$	$\bar{m} = \frac{1}{3-1} \cdot \int_1^3 (-x^2 + 16) dx = \frac{1}{2} \cdot \left[ -\frac{1}{3}x^3 + 16x \right]_{-1}^1 = \frac{1}{2} \cdot \frac{70}{3}$ $= \frac{70}{6} = \mathbf{11, \bar{6}}$
$f(x) = \frac{1}{4}x^4 - 2x^3$ $I = [-1;5]$	$\bar{m} = \frac{1}{5-(-1)} \cdot \int_{-1}^5 \left( \frac{1}{4}x^4 - 2x^3 \right) dx = \frac{1}{6} \cdot \left[ \frac{1}{20}x^5 - \frac{1}{2}x^4 \right]_{-1}^5$ $= -\frac{155,7}{6} = \mathbf{-25,95}$
$f(x) = 6x^3$ $I = [-3;3]$	$\bar{m} = \frac{1}{6} \cdot \int_{-3}^3 6x^3 dx = \frac{1}{6} \cdot \left[ \frac{3}{2}x^4 \right]_{-3}^3 = \frac{1}{6} \cdot \left( \frac{243}{2} - \frac{243}{2} \right) = \mathbf{0}$