Lösung zum Ableiten mit der Produktregel

Aufgabe	Rechenweg	Ergebnis
1. $f(x) = (x^3 + 4x^2 - 3x) \cdot (-x^4 + 1)$	$u(x) = x^3 + 4x^2 - 3x$ $\quad u'(x) = 3x^2 + 8x - 3$ $v(x) = -x^4 + 1$ $\quad v'(x) = -4x^3$ **$f'(x) = u(x) \cdot v'(x) + u'(x) \cdot v(x)$** $f'(x) = (x^3 + 4x^2 - 3x) \cdot (-4x^3) + (3x^2 + 8x - 3) \cdot (-x^4 + 1)$ $= (-4x^6 - 16x^5 + 12x^4) + (-3x^6 - 8x^5 + 3x^4 + 3x^2 + 8x - 3)$ $= -7x^6 - 24x^5 + 15x^4 + 3x^2 + 8x - 3$	$f'(x) = -7x^6 - 24x^5 + 15x^4 + 3x^2 + 8x - 3$
2. $f(x) = x^2 \cdot \sin(x)$	$u(x) = x^2$ $\quad u'(x) = 2x$ $v(x) = \sin(x)$ $\quad v'(x) = \cos(x)$ $f'(x) = x^2 \cdot \cos(x) + 2x \cdot \sin(x)$	$f'(x) = x^2 \cdot \cos(x) + 2x \cdot \sin(x)$
3. $f(x) = x^3 \cdot \ln(x),\ x > 0$	$u(x) = x^3$ $\quad u'(x) = 3x^2$ $v(x) = \ln(x)$ $\quad v'(x) = \frac{1}{x}$ $f'(x) = x^3 \cdot \frac{1}{x} + 3x^2 \cdot \ln(x) = x^2 + 3x^2 \cdot \ln(x)$	$f'(x) = x^2 + 3x^2 \cdot \ln(x)$
4. $f(x) = (2x^3 + 6) \cdot e^x$	$u(x) = 2x^3 + 6$ $\quad u'(x) = 3x^2$ $v(x) = e^x$ $\quad v'(x) = e^x$ $f'(x) = (2x^3 + 6) \cdot e^x + 3x^2 \cdot e^x = (2x^3 + 3x^2 + 6) \cdot e^x$	$f'(x) = (2x^3 + 3x^2 + 6) \cdot e^x$

5. $f(x) = (2x^3 - 4x + 1) \cdot \sin(x)$	$u(x) = 2x^3 - 4x + 1$ $u'(x) = 6x^2 - 4$ $v(x) = \sin(x)$ $v'(x) = \cos(x)$ $f'(x) = (2x^3 - 4x + 1) \cdot \cos(x) + (6x^2 - 4) \cdot \sin(x)$	$f'(x) = (2x^3 - 4x + 1) \cdot \cos(x) + (6x^2 - 4) \cdot \sin(x)$
6. $f(x) = \cos(x) \cdot \sin(x)$	$u(x) = \cos(x)$ $u'(x) = -\sin(x)$ $v(x) = \sin(x)$ $v'(x) = \cos(x)$ $f'(x) = \cos(x) \cdot \cos(x) + (-\sin(x)) \cdot \sin(x)$ $= \cos(x)^2 - \sin(x)^2$	$f'(x) = \cos(x)^2 - \sin(x)^2$
7. $f(x) = 10x^4 + (3x^3 + 2x^2) \cdot \sin(x)$	$u(x) = 3x^3 + 2x^2$ $u'(x) = 9x^2 + 4x$ $v(x) = \sin(x)$ $v'(x) = \cos(x)$ $f'(x) = 40x^3 + (3x^3 + 2x^2) \cdot \cos(x) + (9x^2 + 4x) \cdot \sin(x)$	$f'(x) = 40x^3 + (3x^3 + 2x^2) \cdot \cos(x) + (9x^2 + 4x) \cdot \sin(x)$
8. $f(x) = \sqrt{x} \cdot (3x^2 - 6)$	$u(x) = \sqrt{x} = x^{\frac{1}{2}}$ $u'(x) = \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$ $v(x) = 3x^2 - 6$ $v'(x) = 6x$ $f'(x) = \sqrt{x} \cdot 6x + (\frac{1}{2} \cdot \frac{1}{\sqrt{x}}) \cdot (3x^2 - 6)$	$f'(x) = \sqrt{x} \cdot 6x + (\frac{1}{2} \cdot \frac{1}{\sqrt{x}}) \cdot (3x^2 - 6)$
9. $f(x) = \sqrt[3]{x} \cdot \sin(x)$	$u(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$ $u'(x) = \frac{1}{3} \cdot x^{-\frac{2}{3}} = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}}$ $(x \neq 0)$ $v(x) = \sin(x)$ $v'(x) = \cos(x)$ $f'(x) = \sqrt[3]{x} \cdot \cos(x) + (\frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}}) \cdot \sin(x)$	$f'(x) = \sqrt[3]{x} \cdot \cos(x) + (\frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}}) \cdot \sin(x)$

10. $f(x) = \frac{1}{x^2} \cdot \cos(x)$	$u(x) = \frac{1}{x^2} = x^{-2}$ $\quad u'(x) = -2 \cdot x^{-3} = \frac{-2}{x^3}$ $v(x) = \cos(x)$ $\quad v'(x) = -\sin(x)$ $f'(x) = \frac{1}{x^2} \cdot (-\sin(x)) + \frac{-2}{x^3} \cdot \cos(x)$	$f'(x) = \frac{1}{x^2} \cdot (-\sin(x)) + \frac{-2}{x^3} \cdot \cos(x)$
11. $f(x) = \frac{1}{\sqrt{x}} \cdot (4x^5 + 6x^3)$	$u(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$ $\quad u'(x) = -\frac{1}{2} \cdot x^{-\frac{3}{2}}$ $v(x) = 4x^5 + 6x^3$ $\quad v'(x) = 20x^4 + 18x^2$ $f'(x) = x^{-\frac{1}{2}} \cdot (20x^4 + 18x^2) + (-\frac{1}{2} \cdot x^{-\frac{3}{2}}) \cdot (4x^5 + 6x^3)$	$f'(x) = x^{-\frac{1}{2}} \cdot (20x^4 + 18x^2) + (-\frac{1}{2} \cdot x^{-\frac{3}{2}}) \cdot (4x^5 + 6x^3)$
12. $f(x) = 3 \cdot \sin(x) \cdot (5x + 1)$	$u(x) = 3 \cdot \sin(x)$ $\quad u'(x) = 3 \cdot \cos(x)$ $v(x) = 5x + 1$ $\quad v'(x) = 5$ $f'(x) = 3 \cdot \sin(x) \cdot 5 + 3 \cdot \cos(x) \cdot (5x + 1)$ $= 15 \sin(x) + (15x + 3) \cdot \cos(x)$	$f'(x) = 15 \sin(x) + (15x + 3) \cdot \cos(x)$